

ERRATA

October 1, 1988

to

API RP 11L

RECOMMENDED PRACTICE

for

DESIGN CALCULATIONS

for

SUCKER ROD PUMPING SYSTEMS

(Conventional Units)

FOURTH EDITION

JUNE 1, 1988

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AMERICAN PETROLEUM INSTITUTE

Production Department

2535 One Main Place

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NOTE: Paragraphs A10 through A17 were inadvertently left out of the fourth edition of API RP 11L. These missing paragraphs are included in this Errata.

A10. In determining the natural frequency, the velocity of force propagation "a" plays a key role. The theoretical value for this velocity is about 17,000 feet per second. In practice, however, it has been found that lower frequencies normally occur. The speed of sound in long narrow rods is usually somewhat lower than in normal size vessels. Also, the effect of the rod coupling can cause an apparent increase in density resulting in a decrease in propagation velocity. In practice, it has been found that "a" equals about 16,300 feet per second. With this value, the nondimensional pumping speed can be calculated by:

$$\frac{N}{N_o} = \frac{NL}{245,000 F_o}$$

A11. Another dimensionless parameter of importance in describing the behavior of the sucker rod string is the dimensionless rod stretch, F_o/Sk_r . In this parameter, the spring constant, k_r , is involved. For untapered rod strings, it is evaluated by the relation:

$$k_r = \frac{EA}{L}$$

For tapered rod strings, the spring constant is computed from the familiar reciprocal formula:

$$\frac{1}{k_r} = \frac{1}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \dots \right]$$

The complete term, F_o/Sk_r , gives the rod stretch caused by static application of the fluid load as a percentage of the polished rod stroke. For example,

$$\frac{F_o}{Sk_r} = 0.1$$

means that the rod stretch is 10% of the polished rod stroke when the fluid load is statically applied. At very low speeds where static conditions are approached, the dimensionless rod stretch and pump stroke are related as follows:

$$\frac{S_p}{S} = 1 - \frac{F_o}{Sk_r}$$

At higher speeds, this relation breaks down as dynamic effects become more important.

A12. The charts are used to determine the dependent parameters as defined in the report. Three forces of particular importance are determined from the charts. F_1 is the fluid load plus the maximum dynamic effect on the up stroke. F_2 is the dynamic effects during the down stroke taken away from the load on the polished rod. F_3 is a force which will give horsepower when applied to the full stroke length at the speed of the pumping unit.

A13. The surface dynamometer card which is generated in the analog computer is independent of the weight of the rod string involved. The shape of the card will be the same and a different rod load will simply shift the card up or down in relation to the zero line. In calculating all parameters except torque, this can be handled very easily by calculating fluid and dynamic loads independent of the rod load and then adding in the rod load. In the case of

torque, the amount of rod load is important in determining the torque involved in the unit. All torque values were calculated using a rod load of:

$$\frac{W_{rt}}{Sk_r} = 0.3$$

If a rod load different from this is used, a correction must be made as shown in Fig. 4.6.

A14. In the analog model, it has been assumed that the tubing is anchored and no tubing motion occurs during the pumping stroke. If the tubing is unanchored, then a correction must be made for the shortening of pump stroke which will occur. This happens because the tubing shortens when the load is transferred to the rod string and then lengthens during down stroke when the fluid load is transferred back to the tubing. A correction in pump stroke is made by simply calculating the amount of tubing stretch which will occur with the fluid load used in the design and subtracting this amount of stroke from the net plunger stroke at the bottom of the hole. This should give a reasonable correction. It will not be absolutely correct due to the fact that dynamic effects occur in the tubing string as well as in the rod string and these dynamic effects are not taken into account.

A15. An average conventional pumping unit geometry has been used in the simulation. This is a conventional unit with the counterweights in phase with the crank and the tail bearing being over the slow speed shaft at midpoint of the stroke. For units with drastically different geometry from that assumed, the simulation will not be accurate and the values calculated will be more approximate. It is believed, however, that the values for maximum and minimum loads and for pump stroke will be reasonably good. The calculated value for torque will not be even approximately right and a torque calculation must be made by some other method.

A16. Pumping unit motion was assumed based on a medium slip motor. Generally speaking, a higher slip prime mover results in slight decreases in the maximum load and a slight increase in the minimum load but also tends to reduce the subsurface pump stroke; therefore, some error may be introduced in the calculations if a prime mover with a considerably different slip characteristic than a medium slip electric motor is used. Also, the assumption is made that no friction occurs in the stuffing box or in the pump itself. This, of course, is an unreal assumption although the values for friction in this case should be minor and of negligible importance in the design.

A17. One note of caution in using the new design technique should be pointed out. This method of design will generally give values for load and torque that are higher than values calculated by many previously used pumping design formulas. This tendency of previously used formulas to underestimate the loads has frequently been offset to some degree by a tendency to overestimate the well requirements. The new design method should predict actual well loads to a reasonable degree of accuracy if true well conditions are assumed.

Recommended Practice for Design Calculations for Sucker Rod Pumping Systems (Conventional Units)

API RECOMMENDED PRACTICE 11L (RP 11L)
FOURTH EDITION, JUNE 1, 1988

American Petroleum Institute
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NOMENCLATURE

S_p —Bottom Hole Pump Stroke, inches	$1/k_r$ —Elastic Constant—Total Rod String, inches per pound
PD —Pump Displacement, barrels per day	Note: k_r = Spring Constant of the total rod string and represents the load in pounds required to stretch the total rod string one inch.
$PPRL$ —Peak Polished Rod Load, pounds (See Figure 3.1)	Sk_r —Pounds of load necessary to stretch the total rod string an amount equal to the polished rod stroke, S .
$MPRL$ —Minimum Polished Rod Load, pounds (See Figure 3.1)	N_o —Natural Frequency of Straight Rod String, strokes per minute.
PT —Peak Crank Torque, pound inches	N_o' —Natural Frequency of Tapered Rod String, strokes per minute.
$PRHP$ —Polished Rod Horsepower	$1/k_t$ —Elastic Constant—Unanchored Portion of Tubing String, inches per pound
CBE —Counterweight Required, pounds	Note: k_t = Spring Constant of the unanchored tubing and represents the load in pounds required to stretch the unanchored portion of the tubing, between the anchor and the pump, one inch.
H —Net Lift, feet	W —Total Weight of Rods in air, pounds
L —Pump Depth, feet	W_{rf} —Total Weight of Rods in fluid, pounds (See Figure 3.1)
N —Pumping Speed, Strokes Per Minute	F_1 —PPRL Factor (See Figure 3.1)
S —Polished Rod Stroke, inches (See Figure 3.1)	F_2 —MPRL Factor (See Figure 3.1)
D —Plunger Diameter, inches	T —Crank Torque, pound inches
G —Specific Gravity of Produced Fluid	F_3 —PRHP Factor
W_r —Average Unit Weight of Rods in air, pounds per foot (Table 4.1, Column 3)	T_a —Torque Adjustment Constant for values of W_{rf}/Sk_r other than 0.3
E_r —Elastic Constant—Rods, inches per pound foot (Table 4.1, Column 4)	
Note: E_r represents the inches of elongation caused by the application of a load of one pound to a rod one foot in length.	
F_o —Frequency Factor (Table 4.1, Column 5)	
E_t —Elastic Constant—Tubing, inches per pound foot (Table 4.2, Column 5)	
Note: E_t represents the inches of elongation caused by application of a load of one pound to a section of tubing one foot in length.	
F_o —Differential Fluid Load on full plunger area, pounds (See Figure 3.1)	

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This edition covers editorial changes in API RP 11L (Third Edition, February 1977): *Recommended Practice for Design Calculations for Sucker Rod Pumping Systems (Conventional Units)*, approved at the 1978 Standardization Conference as reported in Circ PS-1589.

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SECTION 1

INTRODUCTION

1.1 In 1954 a group of users and manufacturers of sucker rod pumping equipment undertook a study in depth of the many complex problems associated with this means of lifting fluid from a well. To control and direct the effort, Sucker Rod Pumping Research, Incorporated, a non-profit organization was created. The services of Midwest Research Institute at Kansas City were retained to perform the work necessary to achieve the objectives of the organization.

1.2 The design calculations are based on correlations of the test data that were obtained during the research phase of the project. Sucker Rod Pumping Research, Incorporated, before its dissolution, released these correlated test results to American Petroleum Institute for publication. This *Recommended Practice for the Design Calculations of Sucker Rod Pumping Systems* using conventional pumping units is based on these correlations.

1.3 Three discussions included in the final reports of test results by Midwest Research Institute have been published for permanent reference in *API Drilling and Production Practice (1968)*, p. 232 under the title "Electric Analog Study of Sucker-rod Pumping Systems." These discussions include the following topics:

- a. Vibration Characteristics of Sucker-rod Strings
- b. Physical Characteristics of Sucker Rods
- c. Dimensional Analysis of Sucker-rod Pumping Systems

1.4 A catalog of over 1100 dynamometer cards derived from the electronic analog computer for many combinations of the independent non-dimensional parameters F_0/Sk_r and N/N_0 was included in the material released to API by Sucker Rod Pumping Research, Inc. This catalog has been printed as Bulletin 11L2 and is available from API Production Department, Dallas, Texas.

1.5 Two computer programs have been developed from the data in RP 11L and purchased by API. One program developed tabular material calculated for depths of 2000 ft to 12,000 ft in increments of

500 ft and for production rates of 100 barrels per day to over 1500 barrels per day in varying increments. Rod and pump size combinations as listed in Table 1 of RP 11L were used, except for the elimination of rods 88 and 99. All API stroke lengths are covered. This material is printed as Bulletin 11L3 and is available from API Production Department, Dallas, Texas, 75201.

The other program developed a series of curves for selecting beam pumping units for depths of 1600 ft to 9900 ft and various rates of production and combinations of rod sizes, pump sizes, and speeds. Generally, the limiting factor on the curve is the peak torque rating of the unit. This material is printed as Bulletin 11L4 and is also available from API Production Department, Dallas, Texas.

1.6 American Petroleum Institute (API) Recommended Practices are published to facilitate the broad availability of proven, sound engineering and operating practices. These Recommended Practices are not intended to obviate the need for applying sound judgment as to when and where these Recommended Practices should be utilized.

1.7 The formulation and publication of API Recommended Practices is not intended to, in any way, inhibit anyone from using any other practices.

1.8 Any Recommended Practice may be used by anyone desiring to do so, and a diligent effort has been made by API to assure the accuracy and reliability of the data contained herein. However, the Institute makes no representation, warranty, or guarantee in connection with the publication of any Recommended Practice and hereby expressly disclaims any liability or responsibility for loss or damage resulting from its use, for any violation of any federal, state or municipal regulation with which an API recommendation may conflict, or for the infringement of any patent resulting from the use of this publication.

SECTION 2

VALIDITY OF CALCULATIONS

2.1 In a large majority of cases, it has been found that the values calculated by the following method have been in reasonably close agreement with measured values. Several groups, conducting independent surveys, have found this design method to give better results than other methods formerly used. However, since this method is based on the best interpretations of average values, the actual conditions found in individual cases may not always yield valid predictions of pumping system performance.

2.2 The designer must realize that there are a number of unusual conditions which may be present

in a well that could cause misleading conclusions from these design calculations. Some of these unusual conditions are:

- a. Slanted or crooked holes,
- b. Very viscous fluid,
- c. Excessive sand production,
- d. Excessive gas production through the pump, and
- e. Well flowing-off.

2.3 The research work was limited to simulated problems in which the tubing was assumed as being

anchored at the pump. Therefore, the test results reflected only this condition. However, because of the many known cases in which tubing is unanchored, a formula is included which, experience indicates, will give a very close approximation of relative plunger travel with respect to the pump. This value is identified with the symbol S_p . Examination of the formula will reveal that the contraction of the tubing caused by the transfer of the fluid load from the standing valve to the traveling valve is subtracted from the calculated plunger stroke. It is realized that this formula is highly simplified and not mathe-

matically correct, but it is close enough for practical application.

2.4 These design calculations may be used with confidence when applied to the broad category of average, normal pumping wells fitting the assumed conditions outlined in Appendix A. Unusual conditions not fitting the assumptions will cause deviations from calculated performance. The designer must recognize this fact even though he cannot calculate quantitative values for this deviation.

SECTION 3 DISCUSSION

3.1 An understanding of the formulas utilized for the solution of sucker rod pumping problems will be gained by referring to Figure 3.1. The variables F_0 , F_1 , F_2 , W_{rt} , and S are illustrated with this figure.

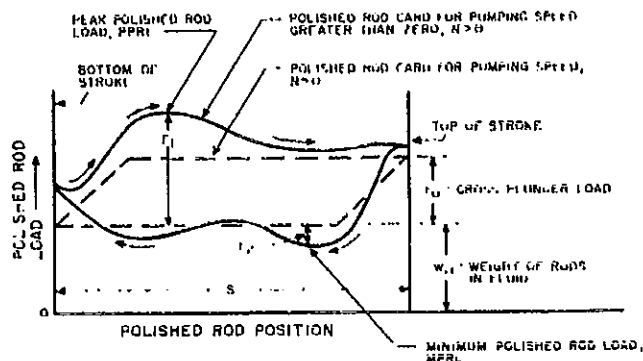


FIGURE 3.1
BASIC DYNAGRAPH CARD

a. At Pumping Speed, $N \cong 0$

Peak Polished Rod Load, $PPRL = W_{rt} + F_0$
Minimum Polished Rod Load, $MPRL = W_{rt}$

b. For Pumping Speed, $N > 0$

Peak Polished Rod Load, $PPRL = W_{rt} + F_1$
Minimum Polished Rod Load, $MPRL = W_{rt} - F_2$

3.2 The problem is generalized by using parameters of variables that are nondimensional.

a. The independent nondimensional variables are:

N/N_0 (Dimensionally = $SPM/SPM = 1$), and
 F_0/Sk_r (Dimensionally = $\frac{\text{Pounds}}{\text{Inches} \times \text{Pounds/Inch}} = 1$)

Where:

$N = SPM$

$N_0 = SPM$ at natural frequency of rod string

$S =$ Surface stroke

$k_r =$ Spring constant of rod string

b. The dependent nondimensional variables are:

Peak Polished Rod Load, $PPRL: F_1/Sk_r$
Minimum Polished Rod Load, $MPRL: F_2/Sk_r$
Peak Torque, $PT: 2T/S^2k_r$
Polished Rod Horsepower, $PRHP: F_3/Sk_r$
Plunger Stroke, $S_p: S_p/S$

3.3 In the research project the sucker rod pumping system was simulated by an electronic analog computer. Computer runs were made for many combinations of N/N_0 and F_0/Sk_r , with the dependent nondimensional variables being measured on each test. Test results were correlated by R. D. Schropp of Phillips Petroleum Company by plotting the families of curves shown in Fig. 4.1 through 4.5. From these curves, values for the various non-dimensional variables may be determined for substitution in the following design calculation formulas:

Plunger Stroke,

$$S_p = [(S_p/S) \times S] - [F_0 \times 1/k_r]$$

NOTE: When tubing is anchored, the value of $1/k_r$ equals zero, therefore the formula for S_p with anchored tubing becomes $(S_p/S) \times S$.

Pump Displacement,

$$PD = 0.1166 \times S_p \times N \times D^2$$

Peak Polished Rod Load,

$$PPRL = W_{rt} + [(F_1/Sk_r) \times Sk_r]$$

Minimum Polished Rod Load,

$$MPRL = W_{rt} - [(F_2/Sk_r) \times Sk_r]$$

Peak Torque,

$$PT = (2T/S^2k_r) \times Sk_r \times S/2 \times T_a$$

Polished Rod Horsepower,

$$PRHP = (F_3/Sk_r) \times Sk_r \times S \times N \times 2.53 \times 10^{-6}$$

Counterweight required,

$$CBE = 1.06 (W_{rt} + \frac{1}{2} F_0)$$

SECTION 4

DESIGN PROCEDURE

4.1 The final solution to this design problem is reached through trial and error methods. Generally, three steps are required in designing an installation.

- a. A preliminary selection of components for the installation must be made.
- b. The operating characteristics of the preliminary selection are calculated by use of the formulas, tables, and figures presented herein.
- c. The calculated pump displacement and loads are compared with the volumes, load ratings, stresses, and other limitations of the preliminary selection.

It will usually be found necessary to make more than one calculation to bring the limitations of the various components of the installation into agreement.

4.2 The minimum amount of information which must be known (or assumed) for a particular sucker rod pumping unit installation design calculation must include:

Fluid Level — H , the net lift in feet
 Pump Depth — L , feet
 Pumping Speed — N , strokes per minute
 Length of Surface Stroke — S , inches
 Pump Plunger Diameter — D , inches
 Specific Gravity of the fluid — G
 The nominal tubing diameter and whether it is anchored or hanging free.
 Sucker rod size and design

4.3 With these factors, the designer will be able to calculate the following:

Plunger Stroke — S_p , inches
 Pump Displacement — PD , barrels per day
 Peak Polished Rod Load — $PPRL$, pounds
 Minimum Polished Rod Load — $MPRL$, pounds
 Peak Crank Torque — PT , pound inches
 Polished Rod Horsepower — $PRHP$
 Counterweight required — CBE , pounds

4.4 Accumulate the known (or assumed) factors on API Form 11L-1 or similar sheet. An example of a completed design calculations form is included on page 6. Form 11L-1 may be obtained in 100-sheet pads by ordering from the API Production Department office in Dallas at a cost of \$2.00 per pad.

4.5 Refer to Table 4.1, use the sucker rod string designation in Column 1 and the plunger diameter in Column 2 as guides, read and record the values for W_r , E_r , and F_r found in Columns 3, 4, and 5 respectively. Table 4.2, Column 5 will give the value of E_t . This factor becomes significant only when working with an unanchored tubing string. If the tubing is anchored, E_t need not be recorded.

NOTE: The values of rod percentages, rod weights, elastic constants, and frequency factors shown in Table 4.1 differ from those in previous editions of RP11L and those shown in Tables 1 and 2, API Bulletin 11L3, First Edition, May 1970. Values in current Table 4.1 were adopted at the June 1976 Standardization Conference, based on the article Sucker Rod String Design by A. B. Neely, published in PETROLEUM ENGINEER, March 1976. Changed rod percentages have negligible effect upon values calculated in Bul 11L3 except for the weight of rods in fluid (W_r). A supplement to the first edition of Bul 11L3 has been published which contains an explanation of the differences and an example for correcting values in Bul 11L3 based on the new rod percentages in Table 4.1.

4.6 Perform the indicated mathematical operations indicated through step 11. If the tubing is anchored, $1/k_t$ (step 11) is equal to zero and need not be calculated. The values are now available with which the bottom hole pump stroke, S_p , and the pump displacement, PD , may be calculated.

4.7 With the calculated values of F_o/Sk_r and N/N_o' record the value of S_p/S from Figure 4.1 and solve for S_p and PD in steps 13 and 14. Pump displacement is the first test being made to see if the preliminary selection of components for the installation is satisfactory. If the pump displacement calculated in step 14 fails to satisfy known or anticipated requirements, appropriate adjustments must be made in the assumed data and steps 1 through 14 repeated. When the calculated pump displacement is acceptable, proceed with the design calculations by performing steps 15, 16, and 17.

4.8 By using the calculated values of F_o/Sk_r and N/N_o' , the values of F_1/Sk_r (Figure 4.2), F_2/Sk_r (Figure 4.3), $2T/S^2k_r$ (Figure 4.4), and F_3/Sk_r (Figure 4.5) are read from the curves and recorded. When referring to Figures 4.1 and 4.6 to determine S_p/S and T_a , the value of N/N_o' must be used. Record the value of T_a .

4.9 Substitution of the appropriate values in the various formulas and performance of the indicated mathematics in steps 23 through 27 will yield the various loads to be expected from the preliminary selection of equipment. It is now necessary to compare these calculated loads with limitations imposed by the preliminary selection. Calculate the stress in the sucker rods to determine if it is within acceptable limits.

4.10 Generally, more than one selection of equipment and calculation of operating conditions is necessary before the optimum selection can be made.

EXAMPLE DESIGN CALCULATIONS CONVENTIONAL SUCKER ROD PUMPING SYSTEM

Object: To solve for— S_p , PD, PPRL, MPRL, PT, PRHP, and CBE

Known or Assumed Data:

Fluid Level, $H = 4,500$ ft. Pumping Speed, $N = 16$ SPM Plunger Diameter, $D = 1.50$ in.
 Pump Depth, $L = 5,000$ ft. Length of Stroke, $S = 54$ in. Spec. Grav. of Fluid, $G = 0.9$
 Tubing Size 2 in. Is it anchored? Yes, ☒ No ☐ Sucker Rods $33.8\% - \frac{7}{8}"$ & $66.2\% - \frac{3}{4}"$

Record Factors from Tables 4.1 & 4.2:

1. $W_r = 1.833$ (Table 4.1, Column 3)
2. $E_r = .804 \times 10^{-6}$ (Table 4.1, Column 4)
3. $F_c = 1.082$ (Table 4.1, Column 5)
4. $E_t = .307 \times 10^{-6}$ (Table 4.2, Column 5)

Calculate Non-Dimensional Variables:

5. $F_o = .340 \times G \times D^2 \times H = .340 \times 0.9 \times 2.25 \times 4,500 = 3,098$ lbs.
6. $1/k_r = E_r \times L = .804 \times 10^{-6} \times 5,000 = 4.020 \times 10^{-3}$ in/lb.
7. $Sk_r = S + 1/k_r = 54 + 4.020 \times 10^{-3} = 13,433$ lbs.
8. $F_o/Sk_r = 3,098 / 13,433 = .231$
9. $N/N_o = NL + 245,000 = 16 \times 5,000 + 245,000 = .326$
10. $N/N_o' = N/N_o \div F_c = .326 \div 1.082 = .301$
11. $1/k_t = E_t \times L = .307 \times 10^{-6} \times 5,000 = 1.535 \times 10^{-3}$ in/lb.

Solve for S_p and PD:

12. $S_p/S = .86$ (Figure 4.1)
13. $S_p = [(S_p/S) \times S] - [F_o \times 1/k_t] = [.86 \times 54] - [3,098 \times 1.535 \times 10^{-3}] = 41.7$ in.
14. $PD = 0.1166 \times S_p \times N \times D^2 = 0.1166 \times 41.7 \times 16 \times 2.25 = 175$ barrels per day

If the calculated pump displacement fails to satisfy known or anticipated requirements, appropriate adjustments must be made in the assumed data and steps 1 through 14 repeated. When the calculated pump displacement is acceptable, proceed with the Design Calculation.

Determine Non-Dimensional Parameters:

15. $W = W_r \times L = 1.833 \times 5,000 = 9,165$ lbs.
16. $W_{rt} = W[1 - (.128G)] = 9,165 [1 - (.128 \times .9)] = 8,110$ lbs.
17. $W_{rt}/Sk_r = 8,110 / 13,433 = .604$

Record Non-Dimensional Factors from Figures 4.2 through 4.6:

18. $F_1/Sk_r = .465$ (Figure 4.2)
19. $F_2/Sk_r = .213$ (Figure 4.3)
20. $2T/S^2k_r = .37$ (Figure 4.4)
21. $F_3/Sk_r = .29$ (Figure 4.5)
22. $T_a = .997$ (Figure 4.6)

Solve for Operating Characteristics:

23. $PPRL = W_{rt} + [(F_1/Sk_r) \times Sk_r] = 8,110 + [.465 \times 13,433] = 14,356$ lbs.
24. $MPRL = W_{rt} - [(F_2/Sk_r) \times Sk_r] = 8,110 - [.213 \times 13,433] = 5,249$ lbs.
25. $PT = (2T/S^2k_r) \times Sk_r \times S/2 \times T_a = .37 \times 13,433 \times 2.7 \times .997 = 133,793$ lb inches
26. $PRHP = (F_3/Sk_r) \times Sk_r \times S \times N \times 2.53 \times 10^{-6} = .29 \times 13,433 \times 54 \times 16 \times 2.53 \times 10^{-6} = 8.5$
27. $CBE = 1.06(W_{rt} + 1/2 F_o) = 1.06 \times (8,110 + 1,549) = 10,239$ lbs.

TABLE 4.1
ROD AND PUMP DATA
See Par. 4.5.

1	2	3	4	5	6	7	8	9	10	11
Rod* No.	Plunger Diam., inches <i>D</i>	Rod Weight, lb per ft <i>W_r</i>	Elastic Constant, in. per lb ft <i>E_r</i>	Frequency Factor, <i>F_c</i>	Rod String, % of each size					
					1½	1	¾	¾	½	¼
44	All	0.726	1.990 x 10 ⁻⁶	1.000	100.0
54	1.06	0.908	1.668 x 10 ⁻⁶	1.138	44.6	55.4
54	1.25	0.929	1.633 x 10 ⁻⁶	1.140	49.5	50.5
54	1.50	0.957	1.584 x 10 ⁻⁶	1.137	56.4	43.6
54	1.75	0.990	1.525 x 10 ⁻⁶	1.122	64.6	35.4
54	2.00	1.027	1.460 x 10 ⁻⁶	1.095	73.7	26.3
54	2.25	1.067	1.391 x 10 ⁻⁶	1.061	83.4	16.6
54	2.50	1.108	1.318 x 10 ⁻⁶	1.023	93.5	6.5
55	All	1.135	1.270 x 10 ⁻⁶	1.000	100.0
64	1.06	1.164	1.382 x 10 ⁻⁶	1.229	33.3	33.1	33.5
64	1.25	1.211	1.319 x 10 ⁻⁶	1.215	37.2	35.9	26.9
64	1.50	1.275	1.232 x 10 ⁻⁶	1.184	42.3	40.4	17.3
64	1.75	1.341	1.141 x 10 ⁻⁶	1.145	47.4	45.2	7.4
65	1.06	1.307	1.138 x 10 ⁻⁶	1.098	34.4	65.6
65	1.25	1.321	1.127 x 10 ⁻⁶	1.104	37.3	62.7
65	1.50	1.343	1.110 x 10 ⁻⁶	1.110	41.8	58.2
65	1.75	1.369	1.090 x 10 ⁻⁶	1.114	46.9	53.1
65	2.00	1.394	1.070 x 10 ⁻⁶	1.114	52.0	48.0
65	2.25	1.426	1.045 x 10 ⁻⁶	1.110	58.4	41.6
65	2.50	1.460	1.018 x 10 ⁻⁶	1.099	65.2	34.8
65	2.75	1.497	0.990 x 10 ⁻⁶	1.082	72.5	27.5
65	3.25	1.574	0.930 x 10 ⁻⁶	1.037	88.1	11.9
66	All	1.634	0.883 x 10 ⁻⁶	1.000	100.0
75	1.06	1.566	0.997 x 10 ⁻⁶	1.191	27.0	27.4	45.6
75	1.25	1.604	0.973 x 10 ⁻⁶	1.193	29.4	29.8	40.8
75	1.50	1.664	0.935 x 10 ⁻⁶	1.189	33.3	33.3	33.3
75	1.75	1.732	0.892 x 10 ⁻⁶	1.174	37.8	37.0	25.1
75	2.00	1.803	0.847 x 10 ⁻⁶	1.151	42.4	41.3	16.3
75	2.25	1.875	0.801 x 10 ⁻⁶	1.121	46.9	45.8	7.2
76	1.06	1.802	0.816 x 10 ⁻⁶	1.072	28.5	71.5
76	1.25	1.814	0.812 x 10 ⁻⁶	1.077	30.6	69.4
76	1.50	1.833	0.804 x 10 ⁻⁶	1.082	33.8	66.2
76	1.75	1.855	0.795 x 10 ⁻⁶	1.088	37.5	62.5
76	2.00	1.880	0.785 x 10 ⁻⁶	1.093	41.7	58.3
76	2.25	1.908	0.774 x 10 ⁻⁶	1.096	46.5	53.5
76	2.50	1.934	0.764 x 10 ⁻⁶	1.097	50.8	49.2
76	2.75	1.967	0.751 x 10 ⁻⁶	1.094	56.5	43.5
76	3.25	2.039	0.722 x 10 ⁻⁶	1.078	68.7	31.3
76	3.75	2.119	0.690 x 10 ⁻⁶	1.047	82.3	17.7
77	All	2.224	0.649 x 10 ⁻⁶	1.000	100.0
85	1.06	1.883	0.873 x 10 ⁻⁶	1.261	22.2	22.4	22.4	33.0
85	1.25	1.943	0.841 x 10 ⁻⁶	1.253	23.9	24.2	24.3	27.6
85	1.50	2.039	0.791 x 10 ⁻⁶	1.232	26.7	27.4	26.8	19.2
85	1.75	2.138	0.738 x 10 ⁻⁶	1.201	29.6	30.4	29.5	10.5

TABLE 4.1 (Continued)
See Par. 4.5.

1	2	3	4	5	6	7	8	9	10	11
Rod* No.	Plunger Diam., inches <i>D</i>	Rod Weight, lb per ft <i>W_r</i>	Elastic Constant, in. per lb ft <i>E_r</i>	Frequency Factor, <i>F_c</i>	Rod String, % of each size					
					1 1/8	1	3/8	1/2	5/8	3/4
86	1.06	2.058	0.742 x 10 ⁻⁶	1.151	22.6	23.0	54.3
86	1.25	2.087	0.732 x 10 ⁻⁶	1.156	24.3	24.5	51.2
86	1.50	2.133	0.717 x 10 ⁻⁶	1.162	26.8	27.0	46.3
86	1.75	2.185	0.699 x 10 ⁻⁶	1.164	29.4	30.0	40.6
86	2.00	2.247	0.679 x 10 ⁻⁶	1.161	32.8	33.2	33.9
86	2.25	2.315	0.656 x 10 ⁻⁶	1.153	36.9	36.0	27.1
86	2.50	2.385	0.633 x 10 ⁻⁶	1.138	40.6	39.7	19.7
86	2.75	2.455	0.610 x 10 ⁻⁶	1.119	44.5	43.3	12.2
87	1.06	2.390	0.612 x 10 ⁻⁶	1.055	24.3	75.7
87	1.25	2.399	0.610 x 10 ⁻⁶	1.058	25.7	74.3
87	1.50	2.413	0.607 x 10 ⁻⁶	1.062	27.7	72.3
87	1.75	2.430	0.603 x 10 ⁻⁶	1.066	30.3	69.7
87	2.00	2.450	0.598 x 10 ⁻⁶	1.071	33.2	66.8
87	2.25	2.472	0.594 x 10 ⁻⁶	1.075	36.4	63.6
87	2.50	2.496	0.588 x 10 ⁻⁶	1.079	39.9	60.1
87	2.75	2.523	0.582 x 10 ⁻⁶	1.082	43.9	56.1
87	3.25	2.575	0.570 x 10 ⁻⁶	1.084	51.6	48.4
87	3.75	2.641	0.556 x 10 ⁻⁶	1.078	61.2	38.8
87	4.75	2.793	0.522 x 10 ⁻⁶	1.038	83.6	16.4
88	All	2.904	0.497 x 10 ⁻⁶	1.000	100.0
96	1.06	2.382	0.670 x 10 ⁻⁶	1.222	19.1	19.2	19.5	42.3
96	1.25	2.435	0.655 x 10 ⁻⁶	1.224	20.5	20.5	20.7	38.3
96	1.50	2.511	0.633 x 10 ⁻⁶	1.223	22.4	22.5	22.8	32.3
96	1.75	2.607	0.606 x 10 ⁻⁶	1.213	24.8	25.1	25.1	25.1
96	2.00	2.703	0.578 x 10 ⁻⁶	1.196	27.1	27.9	27.4	17.6
96	2.25	2.806	0.549 x 10 ⁻⁶	1.172	29.6	30.7	29.8	9.8
97	1.06	2.645	0.568 x 10 ⁻⁶	1.120	19.6	20.0	60.3
97	1.25	2.670	0.563 x 10 ⁻⁶	1.124	20.8	21.2	58.0
97	1.50	2.707	0.556 x 10 ⁻⁶	1.131	22.5	23.0	54.5
97	1.75	2.751	0.548 x 10 ⁻⁶	1.137	24.5	25.0	50.4
97	2.00	2.801	0.538 x 10 ⁻⁶	1.141	26.8	27.4	45.7
97	2.25	2.856	0.528 x 10 ⁻⁶	1.143	29.4	30.2	40.4
97	2.50	2.921	0.515 x 10 ⁻⁶	1.141	32.5	33.1	34.4
97	2.75	2.989	0.503 x 10 ⁻⁶	1.135	36.1	35.3	28.6
97	3.25	3.132	0.475 x 10 ⁻⁶	1.111	42.9	41.9	15.2
98	1.06	3.068	0.475 x 10 ⁻⁶	1.043	21.2	78.8
98	1.25	3.076	0.474 x 10 ⁻⁶	1.045	22.2	77.8
98	1.50	3.089	0.472 x 10 ⁻⁶	1.048	23.8	76.2
98	1.75	3.103	0.470 x 10 ⁻⁶	1.051	25.7	74.3
98	2.00	3.118	0.468 x 10 ⁻⁶	1.055	27.7	72.3
98	2.25	3.137	0.465 x 10 ⁻⁶	1.058	30.1	69.9
98	2.50	3.157	0.463 x 10 ⁻⁶	1.062	32.7	67.3
98	2.75	3.180	0.460 x 10 ⁻⁶	1.066	35.6	64.4
98	3.25	3.231	0.453 x 10 ⁻⁶	1.071	42.2	57.8
98	3.75	3.289	0.445 x 10 ⁻⁶	1.074	49.7	50.3
98	4.75	3.412	0.428 x 10 ⁻⁶	1.064	65.7	34.3
99	All	3.676	0.393 x 10 ⁻⁶	1.000	100.0

TABLE 4.1 (Continued)
See Par. 4.5.

1	2	3	4	5	6	7	8	9	10	11
Rod* No.	Plunger Diam., inches <i>D</i>	Rod Weight, lb per ft. <i>W_r</i>	Elastic Constant, in. per lb ft <i>E_r</i>	Frequency Factor, <i>F_c</i>	Rod String, % of each size					
					1 1/4	1 1/2	1	3/4	5/8	1/2
107	1.06	2.977	0.524 x 10 ⁻⁶	1.184	16.9	16.8	17.1	49.1
107	1.25	3.019	0.517 x 10 ⁻⁶	1.189	17.9	17.8	18.0	46.3
107	1.50	3.085	0.506 x 10 ⁻⁶	1.195	19.4	19.2	19.5	41.9
107	1.75	3.158	0.494 x 10 ⁻⁶	1.197	21.0	21.0	21.2	36.9
107	2.00	3.238	0.480 x 10 ⁻⁶	1.195	22.7	22.8	23.1	31.4
107	2.25	3.336	0.464 x 10 ⁻⁶	1.187	25.0	25.0	25.0	25.0
107	2.50	3.435	0.447 x 10 ⁻⁶	1.174	26.9	27.7	27.1	18.2
107	2.75	3.537	0.430 x 10 ⁻⁶	1.156	29.1	30.2	29.3	11.3
108	1.06	3.325	0.447 x 10 ⁻⁶	1.097	17.3	17.8	64.9
108	1.25	3.345	0.445 x 10 ⁻⁶	1.101	18.1	18.6	63.2
108	1.50	3.376	0.441 x 10 ⁻⁶	1.106	19.4	19.9	60.7
108	1.75	3.411	0.437 x 10 ⁻⁶	1.111	20.9	21.4	57.7
108	2.00	3.452	0.432 x 10 ⁻⁶	1.117	22.6	23.0	54.3
108	2.25	3.498	0.427 x 10 ⁻⁶	1.121	24.5	25.0	50.5
108	2.50	3.548	0.421 x 10 ⁻⁶	1.124	26.5	27.2	46.3
108	2.75	3.603	0.415 x 10 ⁻⁶	1.126	28.7	29.6	41.6
108	3.25	3.731	0.400 x 10 ⁻⁶	1.123	34.6	33.9	31.6
108	3.75	3.873	0.383 x 10 ⁻⁶	1.108	40.6	39.5	19.9
109	1.06	3.839	0.378 x 10 ⁻⁶	1.035	18.9	81.1
109	1.25	3.845	0.378 x 10 ⁻⁶	1.036	19.6	80.4
109	1.50	3.855	0.377 x 10 ⁻⁶	1.038	20.7	79.3
109	1.75	3.867	0.376 x 10 ⁻⁶	1.040	22.1	77.9
109	2.00	3.880	0.375 x 10 ⁻⁶	1.043	23.7	76.3
109	2.25	3.896	0.374 x 10 ⁻⁶	1.046	25.4	74.6
109	2.50	3.911	0.372 x 10 ⁻⁶	1.048	27.2	72.8
109	2.75	3.930	0.371 x 10 ⁻⁶	1.051	29.4	70.6
109	3.25	3.971	0.367 x 10 ⁻⁶	1.057	34.2	65.8
109	3.75	4.020	0.363 x 10 ⁻⁶	1.063	39.9	60.1
109	4.75	4.120	0.354 x 10 ⁻⁶	1.066	51.5	48.5
1010	All	4.538	0.318 x 10 ⁻⁶	1.000	100.00

*Rod No. shown in first column refers to the largest and smallest rod size in eighths of an inch. For example, Rod No. 76 is a two-way taper of 7/8 and 6/8 rods. Rod No. 85 is a four-way taper of 8/8, 7/8, 6/8, and 5/8 rods. Rod No. 109 is a two-way taper of 1 1/4 and 1 1/2 rods. Rod No. 77 is a straight string of 7/8 rods, etc.

TABLE 4.2
TUBING DATA

1	2	3	4	5
Tubing Size	Outside Diameter, in.	Inside Diameter, in.	Metal Area, sq. in.	Elastic Constant, in. per lb ft E_t
1.900	1.900	1.610	0.800	0.500×10^{-6}
2 $\frac{1}{8}$	2.375	1.995	1.304	0.307×10^{-6}
2 $\frac{3}{8}$	2.875	2.441	1.812	0.221×10^{-6}
3 $\frac{1}{8}$	3.500	2.992	2.590	0.154×10^{-6}
4	4.000	3.476	3.077	0.130×10^{-6}
4 $\frac{1}{2}$	4.500	3.958	3.601	0.111×10^{-6}

TABLE 4.3
SUCKER ROD DATA

1	2	3	4
Rod Size	Metal Area, Sq in.	Rod Weight in air, lb per ft W_r	Elastic Constant, in. per lb ft E_r
$\frac{1}{8}$	0.196	0.72	1.990×10^{-6}
$\frac{3}{16}$	0.307	1.13	1.270×10^{-6}
$\frac{1}{4}$	0.442	1.63	0.883×10^{-6}
$\frac{5}{16}$	0.601	2.22	0.649×10^{-6}
1	0.785	2.90	0.497×10^{-6}
1 $\frac{1}{8}$	0.994	3.67	0.393×10^{-6}

TABLE 4.4
PUMP CONSTANTS

1	2	3	4
Plunger Diameter, in. D	Plgr. Diam. Squared D^2	Fluid Load Factor* lb per ft $(.340 \times D^2)$	Pump Factor $(.1166 \times D^2)$
1 $\frac{1}{8}$	1.1289	0.384	0.132
1 $\frac{1}{4}$	1.5625	0.531	0.182
1 $\frac{1}{2}$	2.2500	0.765	0.262
1 $\frac{3}{4}$	3.0625	1.041	0.357
2	4.0000	1.360	0.466
2 $\frac{1}{4}$	5.0625	1.721	0.590
2 $\frac{1}{2}$	6.2500	2.125	0.728
2 $\frac{3}{4}$	7.5625	2.571	0.881
3 $\frac{1}{4}$	14.0625	4.781	1.640
4 $\frac{1}{4}$	22.5625	7.671	2.630

*For fluids with specific gravity of 1.00.

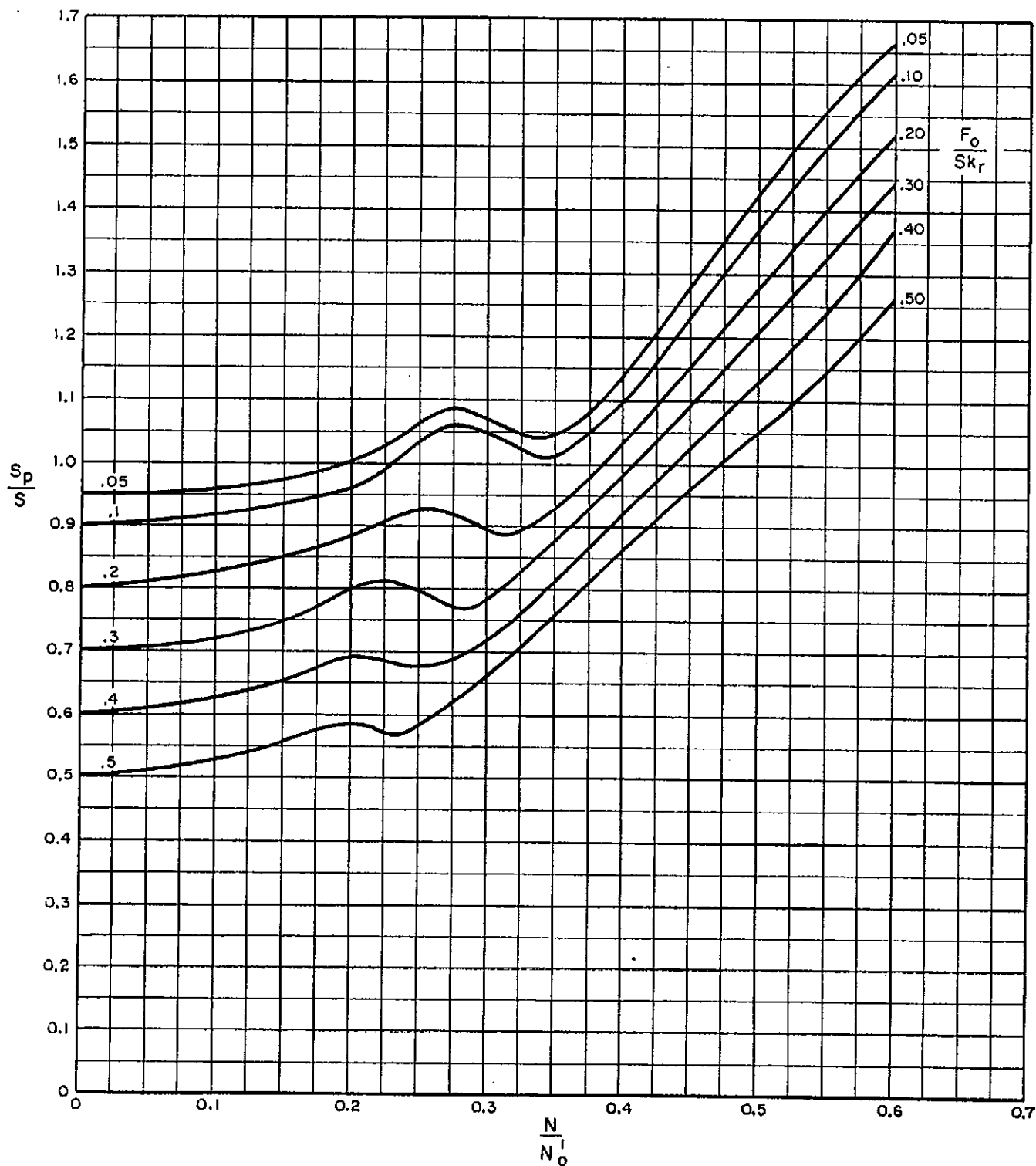


FIGURE 4.1
 $\frac{s_p}{s}$, PLUNGER STROKE FACTOR

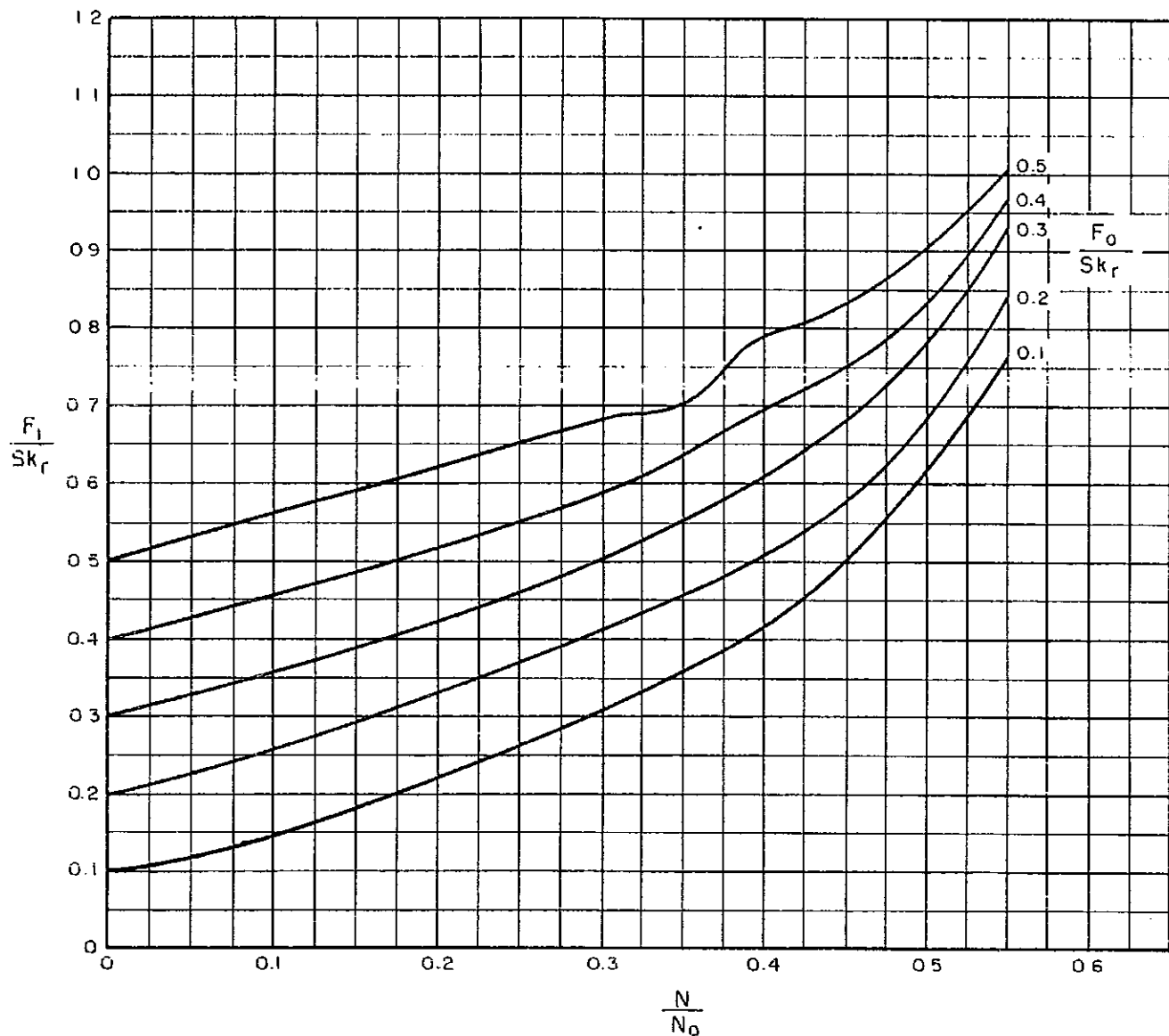


FIGURE 4.2
 $\frac{F_1}{Sk_r}$, PEAK POLISHED ROD LOAD

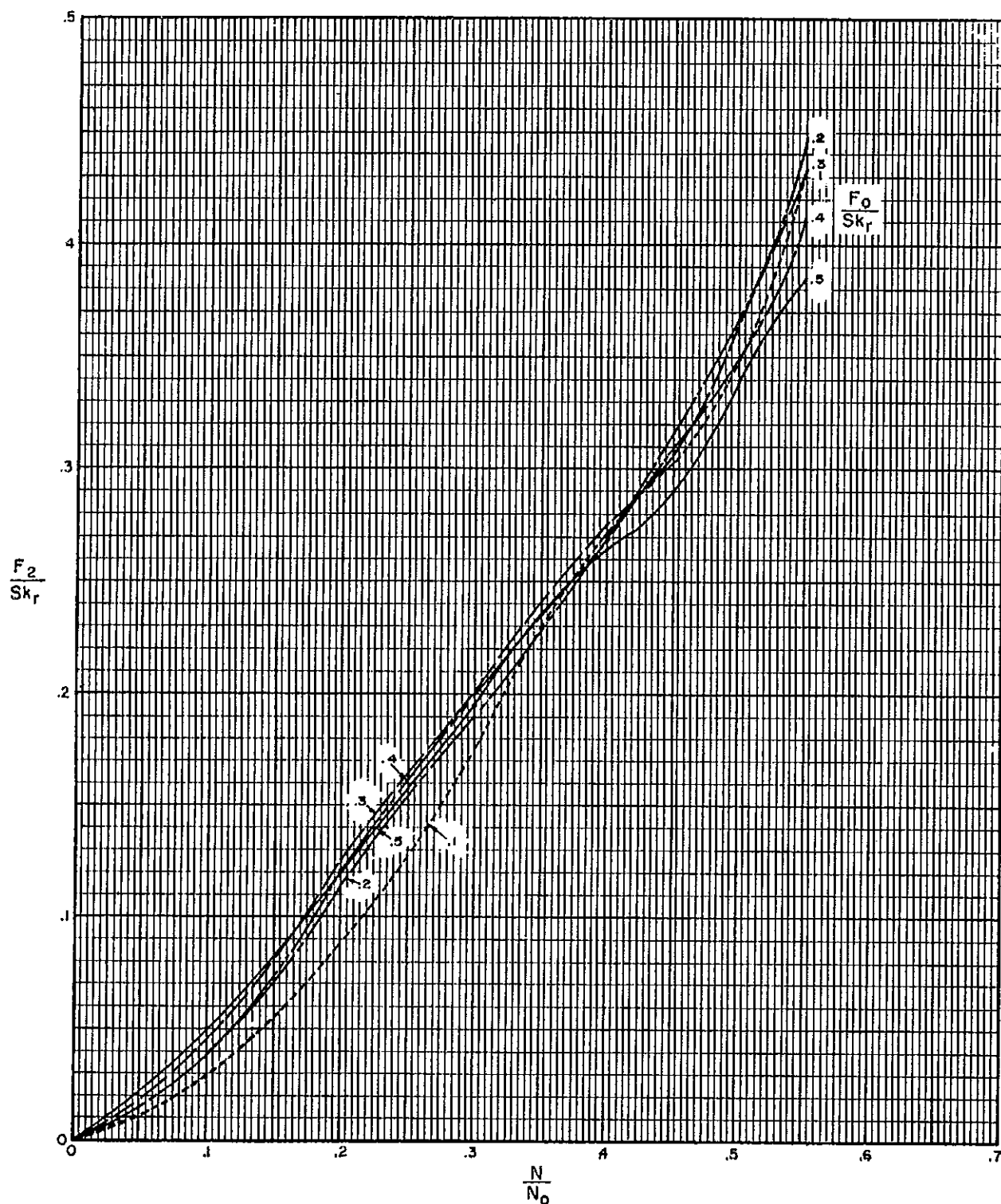
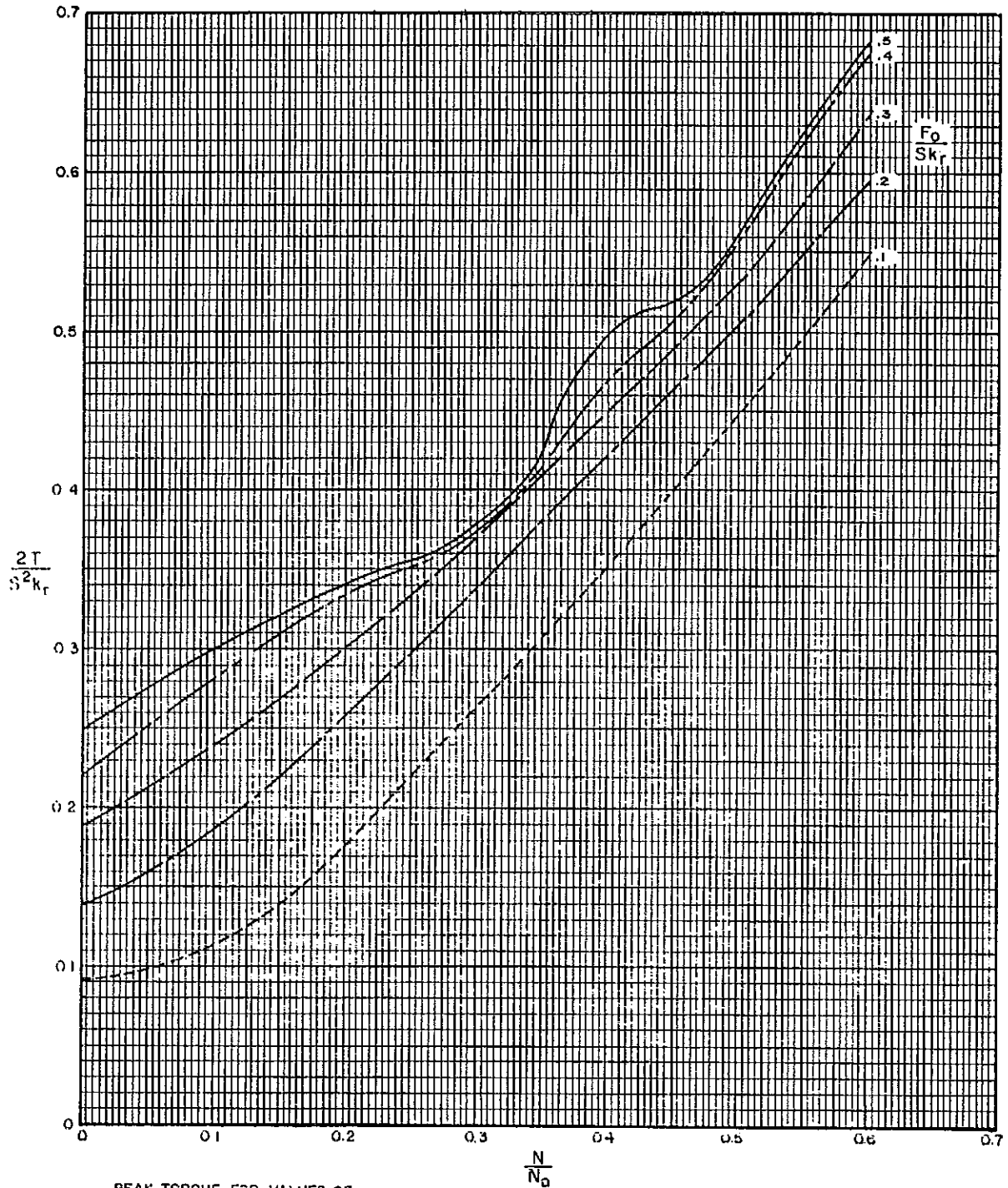


FIGURE 4.3

 $\frac{F_2}{Sk_r}$, MINIMUM POLISHED ROD LOAD



PEAK TORQUE FOR VALUES OF
 $\frac{W_{rf}}{S k_r} = .3$ USE TORQUE ADJUSTMENT
 FOR OTHER VALUES OF $\frac{W_{rf}}{S k_r}$.

FIGURE 4.4
 $\frac{2T}{S^2 k_r}$, PEAK TORQUE

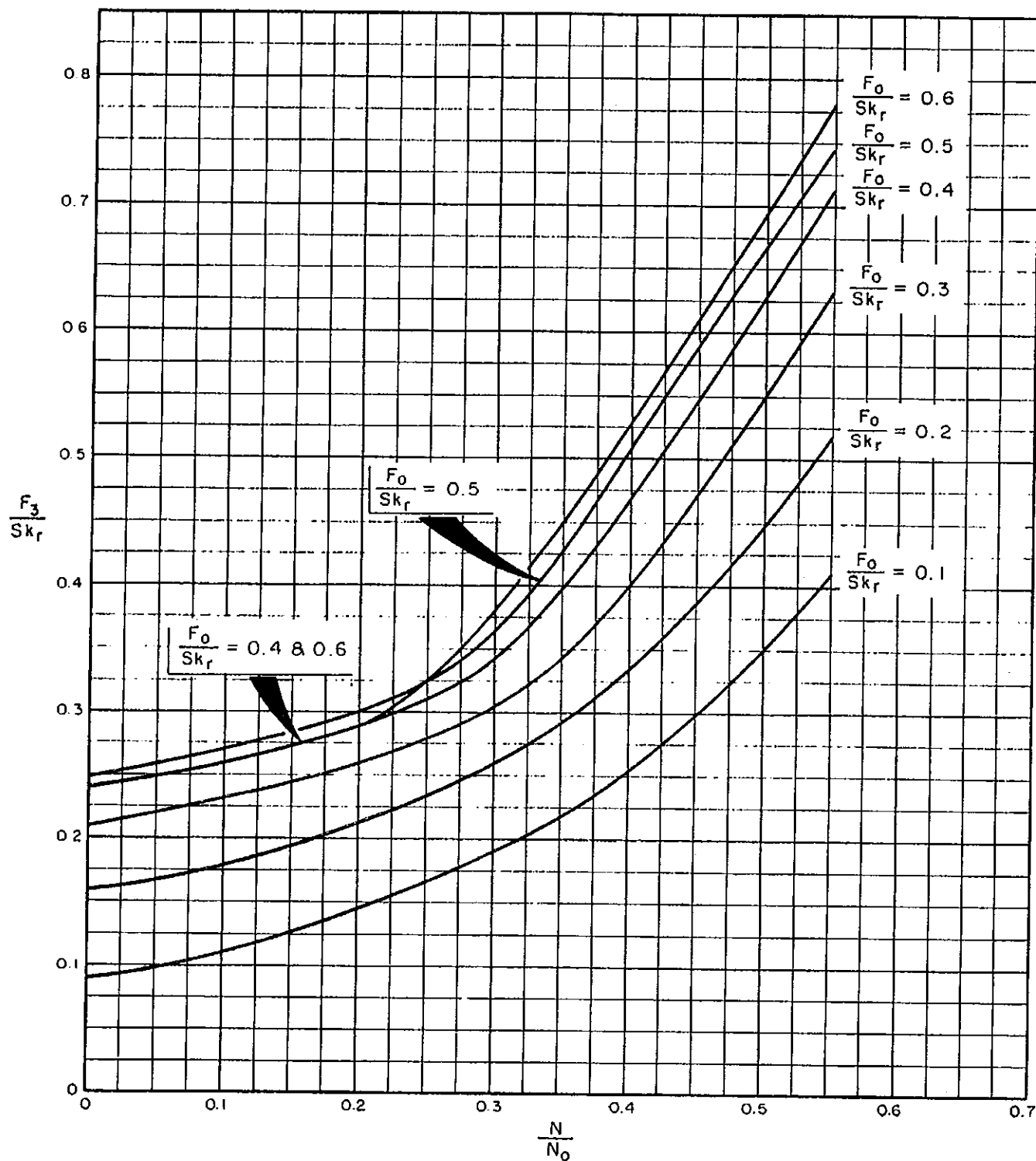
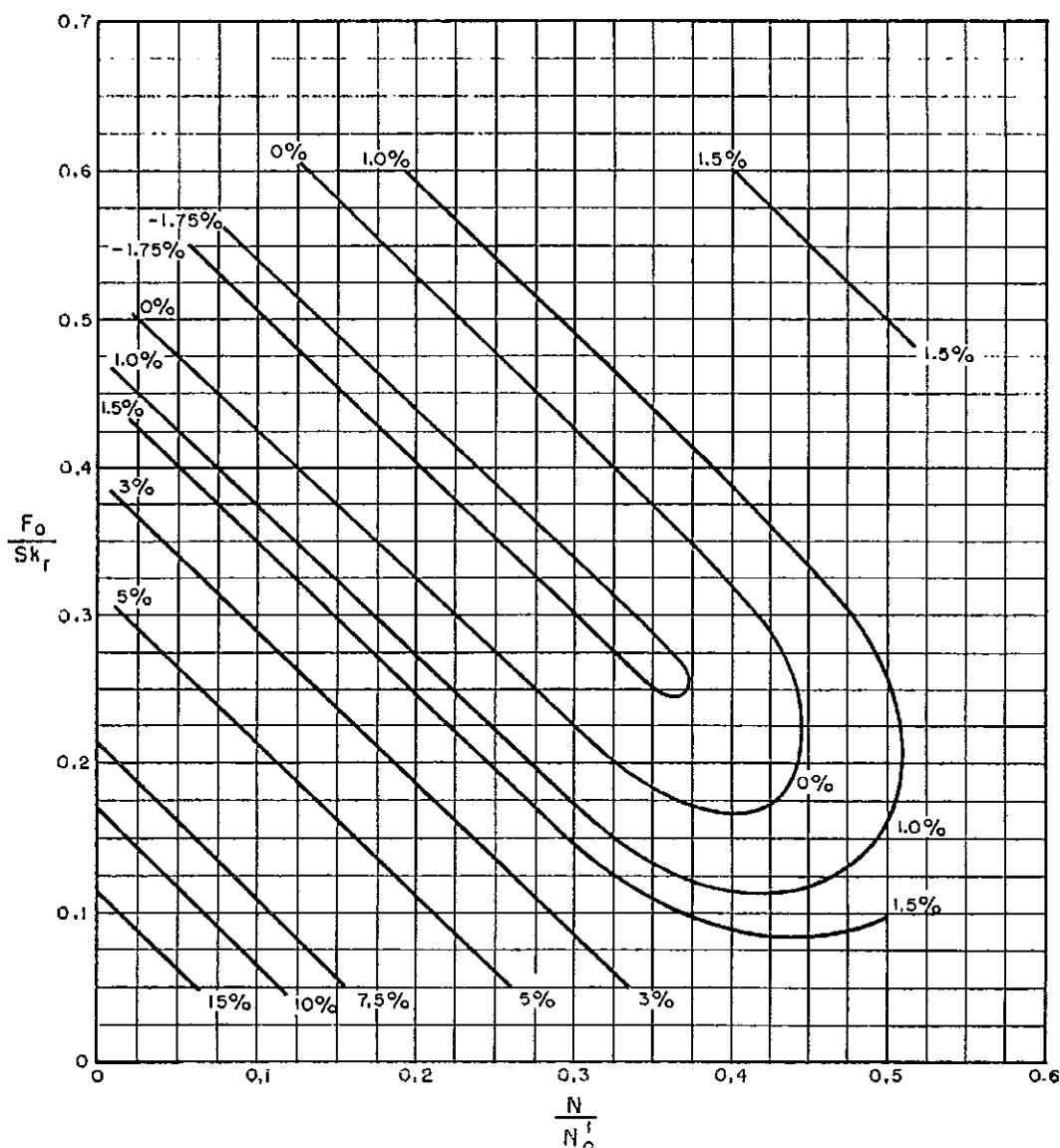


FIGURE 4.5

 $\frac{F_3}{Sk_r}$, POLISHED ROD HORSE POWER



TO USE: MULTIPLY % INDICATED ON CURVE BY $\frac{\left(\frac{W_{rf}}{S_{kr}} - 0.3\right)}{0.1}$

FOR EXAMPLE: $\frac{W_{rf}}{S_{kr}} = 0.600$

$\frac{N}{N_0} = 0.200$ $\frac{F_0}{S_{kr}} = 0.188$

ADJUSTMENT = 3% FOR EACH 0.1 INCREASE IN $\frac{W_{rf}}{S_{kr}}$ ABOVE 0.3

TOTAL ADJUSTMENT = $3 \times 3\% = 9\%$

$T_d = 1.00 + 0.09 = 1.09$

NOTE: IF $\frac{W_{rf}}{S_{kr}}$ IS LESS THAN 0.3 ADJUSTMENT BECOMES NEGATIVE

FIGURE 4.6

T_d , ADJUSTMENT FOR PEAK TORQUE

FOR VALUES OF $\frac{W_{rf}}{S_{kr}}$ OTHER THAN 0.3

APPENDIX A

DISCUSSION OF NONDIMENSIONAL PARAMETERS

A1. A mathematical evaluation of sucker-rod pumping, for practical purposes, revolves around the development of equations to express the motion and the state of stress of sucker rods. The development of representative formulas is mathematically complicated because of the difficulty of expressing boundary conditions which suitably reflect the opening and closing of the valves of the pump when these are themselves a function of the motion of the system and the fluid-flow conditions in the pump.

A2. The criteria generally used for the design of sucker rod installations are based on greatly oversimplified concepts of pumping system mechanics. Consequently, the formulas derived from these viewpoints at best represent average conditions occurring in pumping systems and may not yield valid predictions of system performance, especially in deep wells and at high pumping speeds.

A3. The tendency to oversimplify the methods for predicting system performance is not due to a limited knowledge concerning the operation of sucker rod equipment. Quite the contrary, the basic operating characteristics of sucker rod systems have been reasonably well understood for many years. Rather, oversimplifications in the methods for predicting rod system behavior have arisen as a result of mathematical difficulties and lack of computers to handle the mass calculations required to describe pumping performance accurately.

A4. The simplifications in the mathematical model of sucker rod pumping can be grouped into two broad categories:

- a. Inadequate representation of the rod string.
- b. Inadequate treatment of the downhole pump action.

A5. Many of the current methods have been developed on the basis that the mass of the sucker rod is concentrated at a point. This assumption makes possible a mathematical simplification wherein the spring equation (an ordinary differential equation) is used to simulate the characteristics of the sucker rod. Although this assumption is mathematically convenient, it destroys the analogy between the actual sucker rod and the mathematical model used to represent the sucker rod. In reality, the mass of the sucker rod is distributed along its length, and this fact must be incorporated into any realistic mathematical model of the rod. Without this provision, the analysis does not include the effect of force waves traveling within the sucker rod, which is an important characteristic of real systems.

A6. Some of the later techniques for predicting rod performance have used an adequate representation of the sucker rod. Such a representation requires that the wave equation (a partial differential equation) be used to describe the behavior of the sucker rod. However, use of the wave equation introduces a new mathematical difficulty, in that partial differential equations are inherently more difficult to solve. Much of the difficulty arises in formulating the boundary conditions that describe

the loading and motion at the extremities of the rod. The boundary condition which describes the behavior of the downhole pump presents particular difficulties, and investigators in the past have made many specialized assumptions in its regard.

A7. The apparent necessity for making specialized assumptions about pump action has been brought about by peculiarities in the sucker rod problem. The sucker rod problem presents a particular difficulty, in that the pump boundary condition depends upon the behavior of the sucker rod string itself, which is the very thing to be established by solution of the problem. This apparent impasse has led to the special assumptions mentioned heretofore, which, in effect, amount to idealized guesses about pump operation. Consequently, the methods based on the special assumptions yield usable results only to the extent to which the idealizations approximate pump action.

A8. In the work at Midwest Research Institute, sucker rod pumping systems were simulated by an electronic analog computer. Numerous runs were made representing a wide range of conditions. These runs were correlated on the basis of two nondimensional parameters. These parameters are N/N_o' , the dimensionless pumping speed, and F_o/Skr , the dimensionless rod stretch. The use of these nondimensional parameters allows a complete suite of sucker rod pumping conditions to be correlated without having to run an infinite number of cases.

A9. The dimensionless pumping speed, N/N_o' , is a highly important index of the rod strings' behavior. This parameter is the ratio of the forcing frequency, N (the pumping speed), to the undamped natural frequency of the rod string, N_o' . The undamped natural frequency is shown in vibration theory to be inversely proportional to the time required for a force wave to make four traversals along the sucker rod. Thus, the undamped natural frequency is given by:

$$N_o' = \frac{F_o a}{4L}$$

where F_o is a constant of proportionality which depends on the rod design and "a" is the speed of sound in steel. For untapered rod strings, F_o equals 1. From theoretical considerations, it can be shown that the natural frequency of a tapered rod string is greater than that of a uniform string of equal length. Thus, for tapered strings, F_o has a value greater than unity. Values for F_o in tapered rod strings can be found in Table 4.1. The formula for dimensionless pumping speed N/N_o' immediately follows as:

$$\frac{N}{N_o'} = \frac{4NL}{F_o a}$$

Figures A.1 through A.6 contain charts from which most of the frequency factors, F_o , in Table 4.1 were obtained. These charts may be used to determine F_o for any arrangement of rod sizes for some of the most commonly used tapered strings. Fig. A.6 plots a number of rod combinations on a single curve as a function of pump diameter.

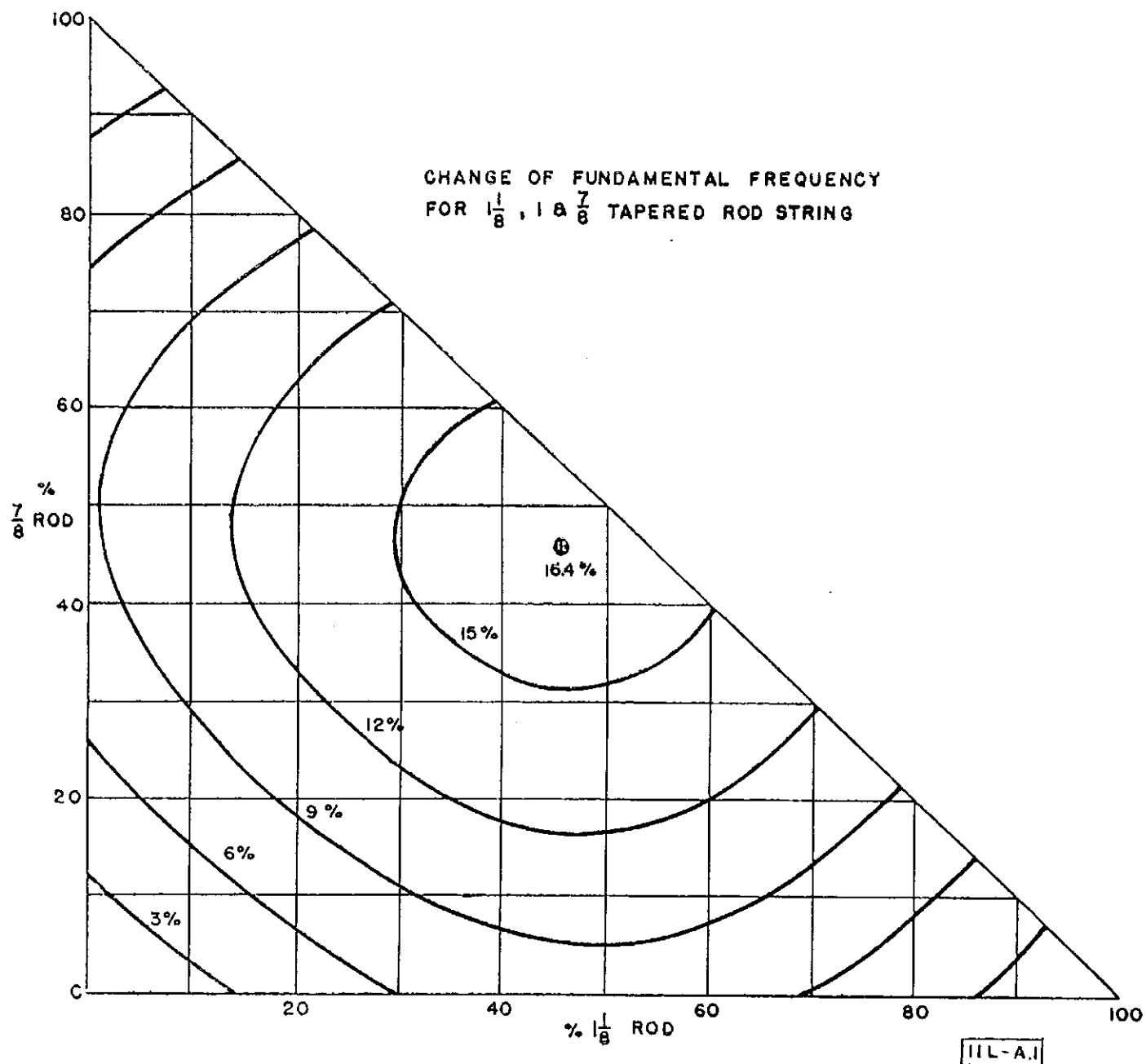


FIG. A.1
PERCENTAGE INCREASE IN FUNDAMENTAL FREQUENCY
1-1/8, 1, AND 7/8-INCH THREE-WAY TAPER STRING

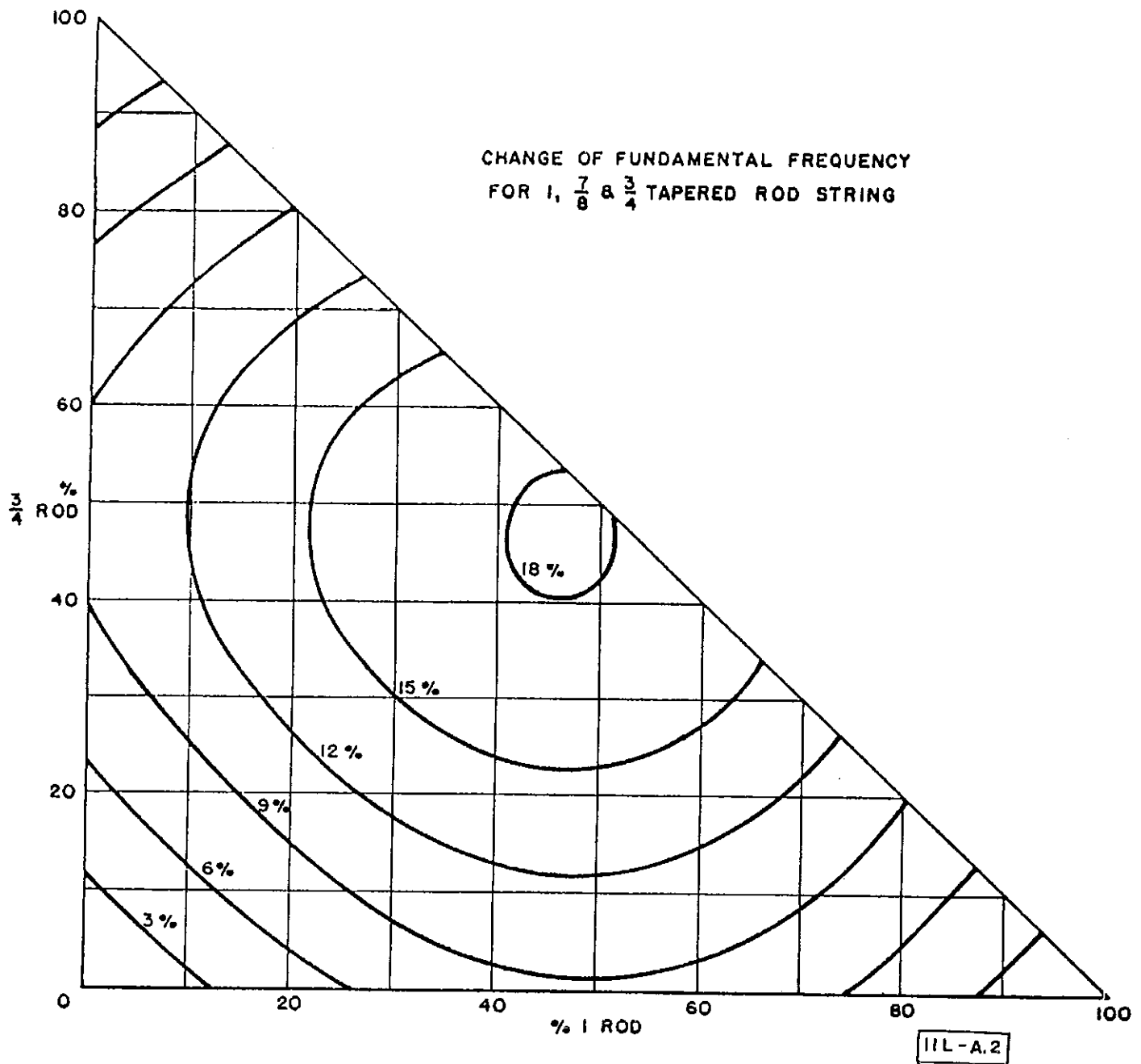


FIG. A2
PERCENTAGE INCREASE IN FUNDAMENTAL FREQUENCY
1, $\frac{7}{8}$, AND $\frac{3}{4}$ -INCH THREE-WAY TAPER STRING

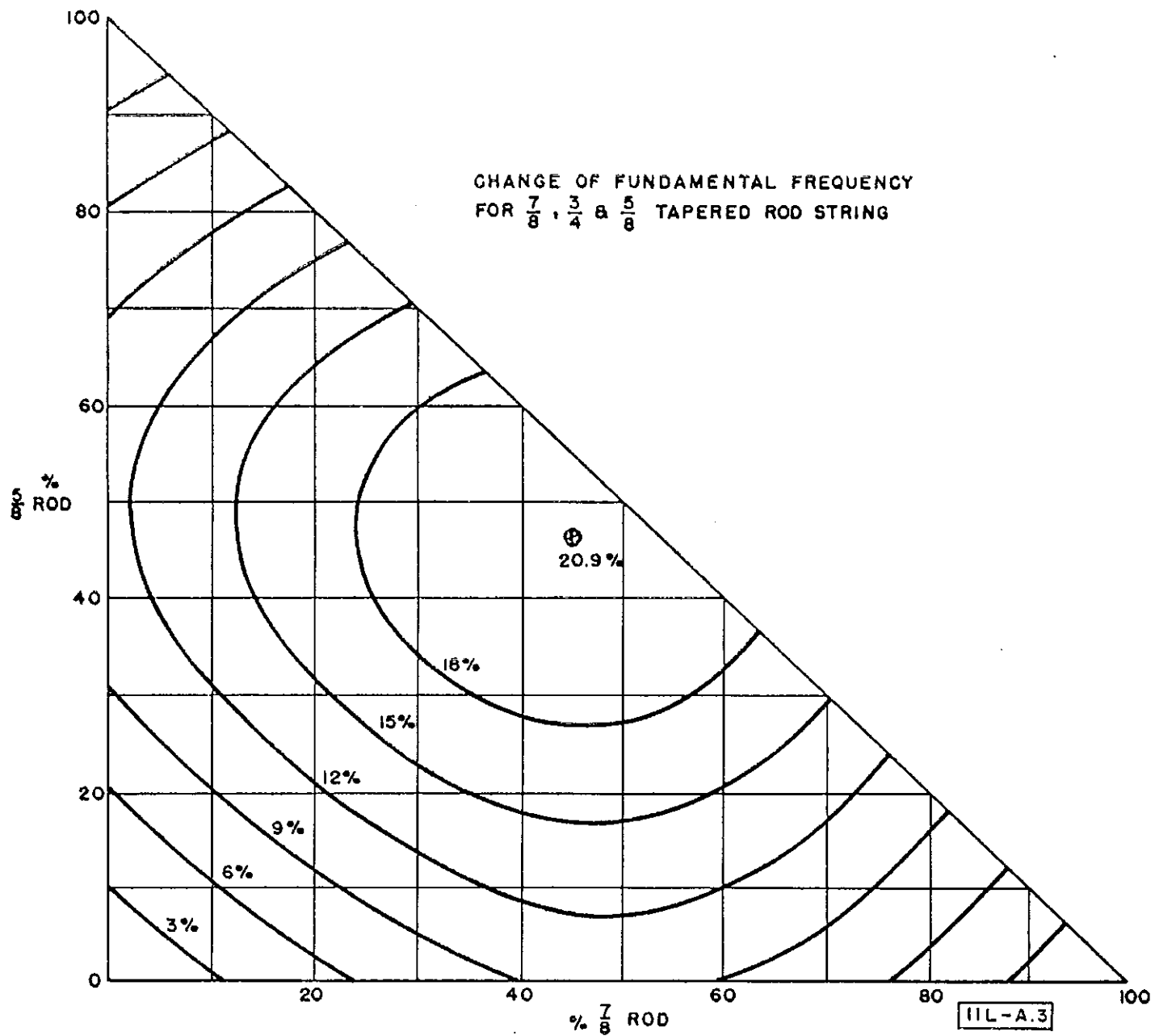


FIG. A.3
PERCENTAGE INCREASE IN FUNDAMENTAL FREQUENCY
 $\frac{7}{8}$, $\frac{3}{4}$, AND $\frac{5}{8}$ -INCH THREE-WAY TAPER STRING

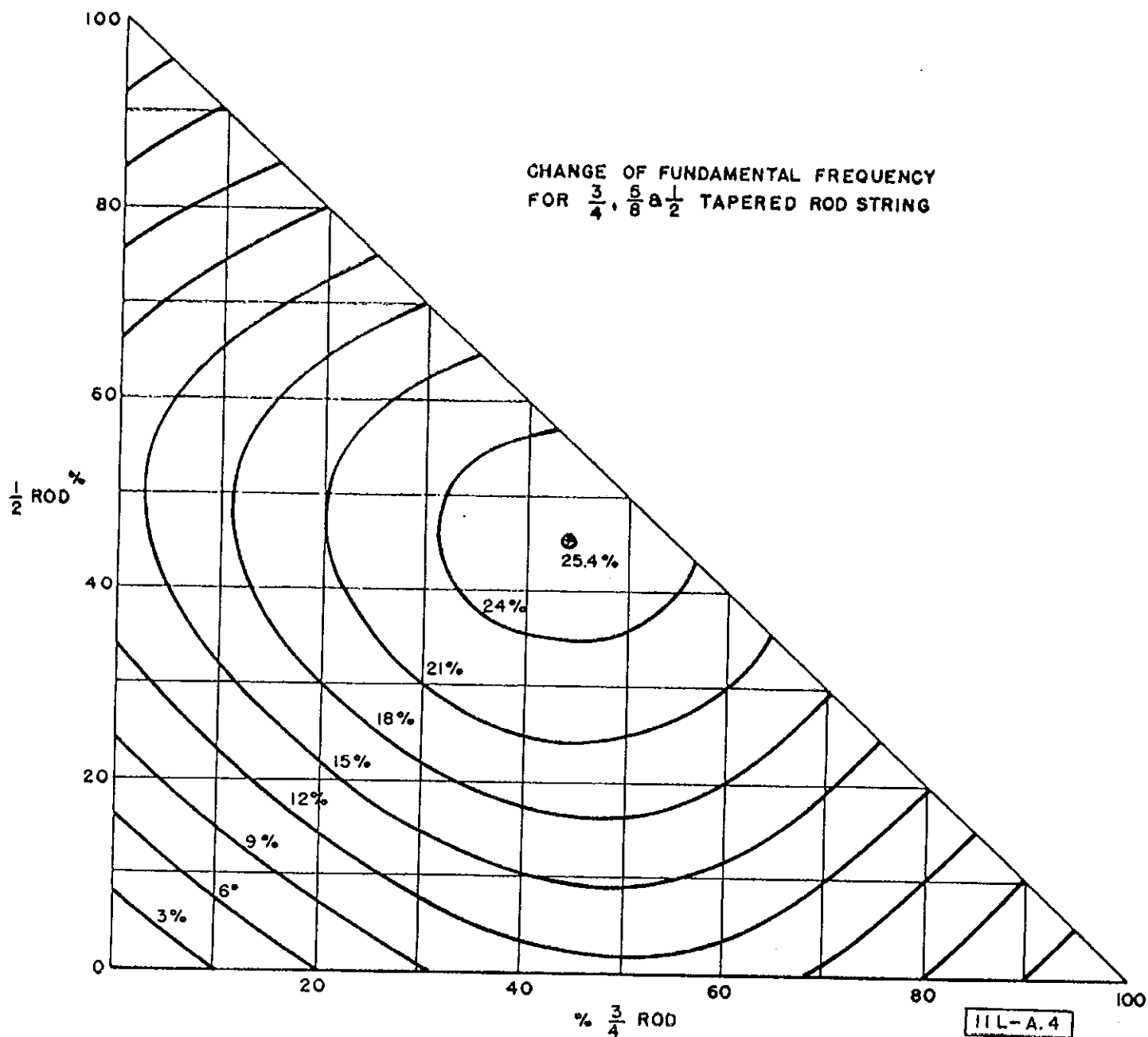


FIG. A.4
PERCENTAGE INCREASE IN FUNDAMENTAL FREQUENCY
 $\frac{3}{4}$, $\frac{5}{8}$, AND $\frac{1}{2}$ -INCH THREE-WAY TAPER STRING

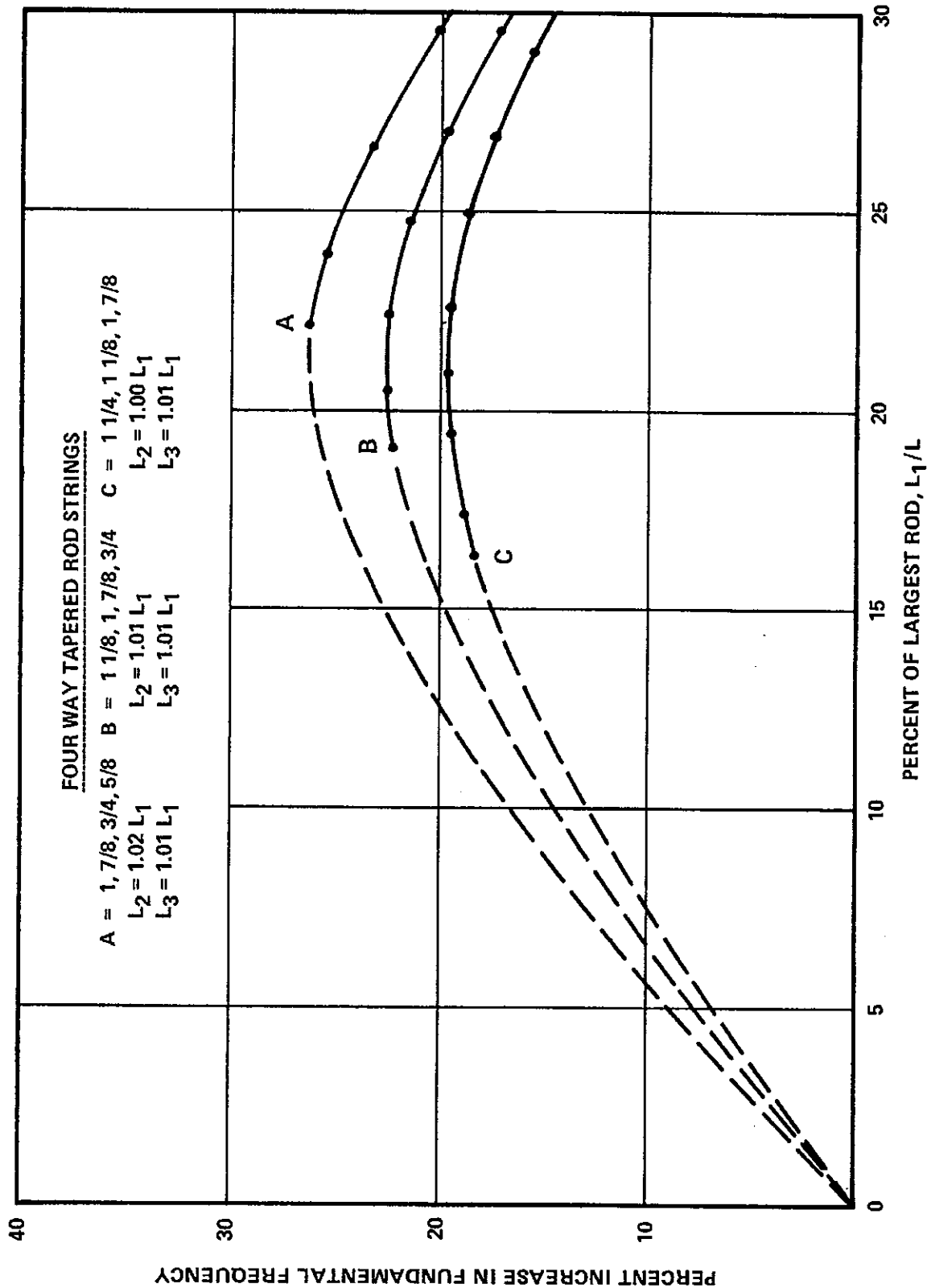


FIGURE A.5
FOUR-WAY TAPER STRINGS
PERCENT INCREASE IN FUNDAMENTAL FREQUENCY

11L-A.5

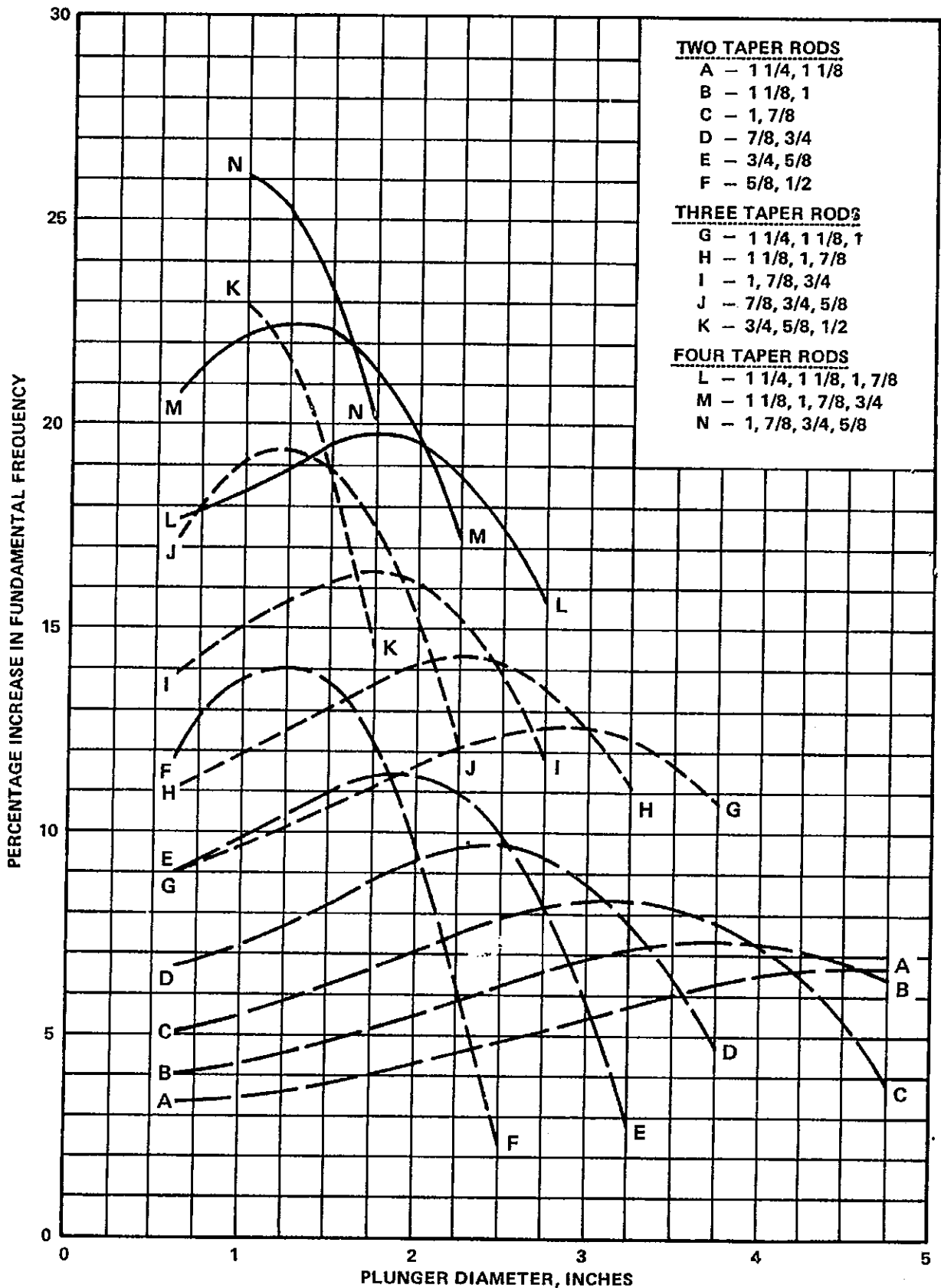


FIGURE A.6
PERCENTAGE INCREASE IN FUNDAMENTAL FREQUENCY
SPECIFIC ROD COMBINATIONS

IIL - A.6